

Heavy Meson Exclusive Decays in the Framework of QCD Sum Rules^{1,*}

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Abstract

We discuss applications of QCD sum rules on the light-cone to the form factors of the exclusive transitions $B \rightarrow \pi$ and $D \rightarrow \pi$, and to the $B^*B\pi$ and $D^*D\pi$ coupling constants. In the light of our results we examine the pole dominance model for these form factors. A first estimate is given on the nonfactorizable amplitude of the decay $B \rightarrow J/\psi K$.

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1 Introduction

The reliable extraction of fundamental parameters from data on heavy flavoured hadrons is an important theoretical task. While the inclusive B and D decays appear to be the cleanest reactions theoretically, exclusive decays are experimentally often more favourable. However, for the interpretation of exclusive measurements one needs an accurate knowledge of decay constants, form factors and other hadronic matrix elements. Among the existing approaches, QCD sum rules [1] have proved to be particularly powerful in obtaining reliable estimates. In this report, we discuss applications of the sum rule method to form factors of the transitions $D \rightarrow \pi$ and $B \rightarrow \pi$, to the $D^*D\pi$ and $B^*B\pi$ coupling constants, and to the nonfactorizable amplitude of the decay $B \rightarrow J/\psi K$. From a more technical point of view, our calculations aim at developing alternative variants of sum rules which avoid some of the problems inherent in the more familiar original version.

As explained in Section 2, the so-called light-cone sum rules provide a very economical way to obtain B and D form factors and couplings. In this variant, the ideas of duality and matching between parton and hadron descriptions intrinsic to QCD sum rules are combined with the operator product expansion (OPE) techniques used to study hard exclusive processes in QCD [2, 3]. Using these results, we then examine the pole dominance model for form factors in Section 3. Finally, in Section 4 we describe an attempt to estimate weak amplitudes beyond the usual factorization approximation considering the decay mode $B \rightarrow J/\psi K$ as a prototype example and employing conventional sum rule methods.

2 Transition form factors and hadronic couplings

2.1 QCD sum rules on the light-cone

In contrast to the conventional sum rules based on the Wilson OPE of the T-product of currents at small distances, one may consider expansions near the light-cone in terms of nonlocal operators, the matrix elements of which are given by hadron wave functions of increasing twist. As one advantage, this formulation allows to incorporate additional information about the Euclidean asymptotics of correlation functions in QCD for arbitrary external momenta.

For definiteness, we focus on the correlation function which will later be used to evaluate the form factor $D \rightarrow \pi$ and the $D^*D\pi$ coupling:

$$\begin{aligned} F_\mu(p, q) &= i \int d^4x e^{ipx} \langle \pi^-(q) | T \{ \bar{d}(x) \gamma_\mu c(x), \bar{c}(0) i \gamma_5 u(0) \} | 0 \rangle \\ &= F(p^2, (p+q)^2) q_\mu + \tilde{F}(p^2, (p+q)^2) p_\mu . \end{aligned} \quad (1)$$

With the pion on mass-shell, $q^2 = m_\pi^2$, the correlation function (1) depends on two invariants, p^2 and $(p+q)^2$. We set $m_\pi = 0$ everywhere.

In the Euclidean region where both p^2 and $(p+q)^2$ are negative and large, the charm quark is far off-shell. Substituting, as a first approximation, the free c -quark propagator

$$\langle 0 | T \{ c(x) \bar{c}(0) \} | 0 \rangle = i \hat{S}_c^0(x) = \int \frac{d^4k}{(2\pi)^4 i} e^{-ikx} \frac{\not{k} + m_c}{m_c^2 - k^2} \quad (2)$$

into eq. (1) one readily obtains

$$F_\mu(p, q) = i \int \frac{d^4x d^4k}{(2\pi)^4 (m_c^2 - k^2)} e^{i(p-k)x} \left(m_c \langle \pi(q) | \bar{d}(x) \gamma_\mu \gamma_5 u(0) | 0 \rangle \right)$$

$$+ k^\nu \langle \pi(q) | \bar{d}(x) \gamma_\mu \gamma_\nu \gamma_5 u(0) | 0 \rangle \rangle . \quad (3)$$

This contribution is depicted diagrammatically in Fig. 1a.

Short-distance expansion of the first matrix element of eq. (3) in terms of local operators,

$$\bar{d}(x) \gamma_\mu \gamma_5 u(0) = \sum_n \frac{1}{n!} \bar{d}(0) (\overleftarrow{D} \cdot x)^n \gamma_\mu \gamma_5 u(0) , \quad (4)$$

and integration over x and k yield

$$F_\mu(p, q) = i \frac{m_c}{m_c^2 - p^2} \sum_{n=0}^{\infty} \frac{(2p \cdot q)^n}{(m_c^2 - p^2)^n} M_n q_\mu , \quad (5)$$

where

$$\langle \pi(q) | \bar{d} \overleftarrow{D}_{\alpha_1} \overleftarrow{D}_{\alpha_2} \dots \overleftarrow{D}_{\alpha_n} \gamma_\mu \gamma_5 u | 0 \rangle = (i)^n q_\mu q_{\alpha_1} q_{\alpha_2} \dots q_{\alpha_n} M_n + \dots$$

has been used, D being the covariant derivative. One now encounters the following problem. If the ratio

$$\tilde{\xi} = 2(p \cdot q)/(m_c^2 - p^2) = ((p + q)^2 - p^2)/(m_c^2 - p^2) \quad (6)$$

is finite one must keep an *infinite* series of matrix elements of local operators in eq. (5). All of them give contributions of the order $1/(m_c^2 - p^2)$ in the heavy quark propagator, differing only by powers of the dimensionless parameter $\tilde{\xi}$. Therefore, short-distance expansion of eq. (3) is useful *only* if $\tilde{\xi} \rightarrow 0$, i.e. for $p^2 \simeq (p + q)^2$ or, equivalently, $q \simeq 0$. In this case, the series in eq. (5) can be truncated after a few terms involving only a small number of unknown matrix elements M_n . However, for general momenta with $p^2 \neq (p + q)^2$ one has to sum up the infinite series of matrix elements of local operators in some way.

This formidable task can be solved by using techniques developed for hard exclusive processes in QCD [2, 3]. Returning to the initial expression (1) for the correlation function one expands the T -product of currents near the light-cone $x^2 = 0$. In a first step this leads to the same approximation (3) involving vacuum-to-pion transition matrix elements of nonlocal operators composed of light quark fields at light-like separation. These matrix elements are expanded in x and at $x^2 \simeq 0$ reexpressed in terms of pion wave functions with given twist. For the present discussion it is again sufficient to focus on the first term in eq. (3) proportional to m_c . In leading twist one has

$$\langle \pi(q) | \bar{d}(x) \gamma_\mu \gamma_5 u(0) | 0 \rangle = -i q_\mu f_\pi \int_0^1 du e^{i u q x} \varphi_\pi(u) , \quad (7)$$

where the wave function φ_π represents the distribution in the fraction u of the light-cone momentum $q_0 + q_3$ of the pion carried by a constituent quark. Substituting eq. (7) in eq. (3) and integrating over x and k one finds for the invariant function F defined in eq. (1):

$$F(p^2, (p + q)^2) = m_c f_\pi \int_0^1 \frac{du \varphi_\pi(u)}{m_c^2 - (p + uq)^2} + \dots , \quad (8)$$

where the ellipses represent contributions of higher twists and multicomponent wave functions. The leading three-particle wave function enters in connection with gluon emission by the heavy quark line as shown in Fig. 1b. This contribution is included in the calculations of refs. [4, 5]

as well as two-particle wave functions up to twist 4. The calculation of perturbative $O(\alpha_s)$ corrections indicated in Fig. 1c and 1d is in progress.

Comparing eqs. (5) and (8) one sees that the infinite series of matrix elements of local operators encountered before in eq. (5) is effectively replaced by hadronic wave functions. These universal functions describe the long-distance dynamics similarly as the universal vacuum condensates appearing in the more familiar sum rule variant based on short-distance expansion. The universality property is essential for the light-cone approach.

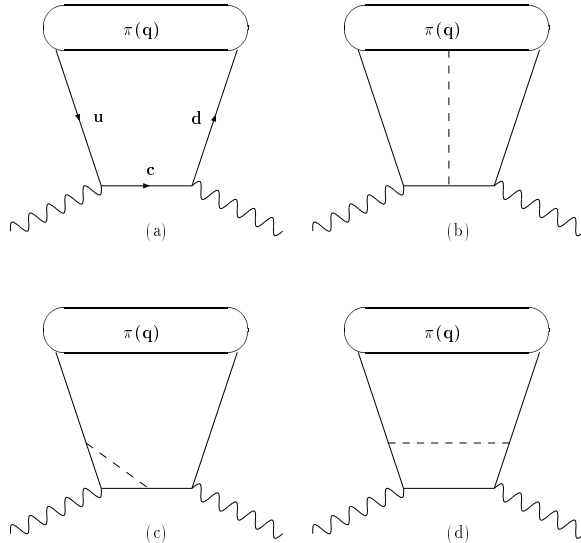


Fig. 1. QCD diagrams contributing to the correlation function (1) and involving (a) quark-antiquark light-cone wave functions, (b) three-particle quark-antiquark-gluon wave functions, (c) and (d) perturbative $O(\alpha_s)$ corrections. Solid lines represent quarks, dashed lines gluons, wavy lines are external currents.

2.2 $D \rightarrow \pi$ and $B \rightarrow \pi$ form factors

The light-cone sum rule for the form factor $f_D^+(p^2)$ entering the transition amplitude

$$\langle \pi(q) | \bar{d}\gamma_\mu c | D(p+q) \rangle = 2f_D^+(p^2)q_\mu + (f_D^+(p^2) + f_D^-(p^2))p_\mu \quad (9)$$

is obtained by matching the expression (8) for the invariant amplitude $F(p^2, (p+q)^2)$ in terms of pion wave functions with the hadronic representation

$$F(p^2, (p+q)^2) = \frac{2m_D^2 f_D f_D^+(p^2)}{m_c(m_D^2 - (p+q)^2)} + \int_{s_0}^{\infty} \frac{\rho^h(p^2, s) ds}{s - (p+q)^2} . \quad (10)$$

In the above, the pole term is due to the ground state in the heavy channel, while the excited and continuum states are taken into account by the dispersion integral with the effective threshold s_0 . Invoking semilocal duality, the latter contributions are cancelled against the

corresponding piece of the dispersion integral representation of the QCD result on the l.h.s. of eq. (10). After Borel transformation in the variable $(p+q)^2$, i.e. after applying the operator

$$\mathcal{B}_{M^2} f(Q^2) = \lim_{Q^2, n \rightarrow \infty, Q^2/n = M^2} \frac{(Q^2)^{(n+1)}}{n!} \left(-\frac{d}{dQ^2} \right)^n f(Q^2) \equiv f(M^2) \quad (11)$$

to eq. (10) where M^2 is called the Borel parameter, one finds the following sum rule:

$$\begin{aligned} f_D f_D^+(p^2) = & \frac{f_\pi m_c^2}{2m_D^2} \left\{ \int_\Delta \frac{du}{u} \exp \left[\frac{m_D^2}{M^2} - \frac{m_c^2 - p^2(1-u)}{uM^2} \right] \Phi_2(u, M^2, p^2) \right. \\ & - \int_0^1 u du \int_0^1 d\alpha_1 \int_0^{1-\alpha_1} d\alpha_2 \frac{\Theta(\alpha_1 + u\alpha_2 - \Delta)}{(\alpha_1 + u\alpha_2)^2} \\ & \times \exp \left[\frac{m_D^2}{M^2} - \frac{m_c^2 - p^2(1-\alpha_1 - u\alpha_2)}{(\alpha_1 + u\alpha_2)M^2} \right] \Phi_3(u, M^2, p^2) \Big\} , \end{aligned} \quad (12)$$

where $\Delta = (m_c^2 - p^2)/(s_0^{(c)} - p^2)$ and

$$\Phi_2 = \varphi_\pi(u) + \frac{\mu_\pi}{m_c} \left[u\varphi_p(u) + \frac{1}{6}\varphi_\sigma(u) \left(2 + \frac{m_c^2 + p^2}{uM^2} \right) \right] + \dots , \quad (13)$$

$$\Phi_3 = \frac{2f_{3\pi}}{f_\pi m_c} \varphi_{3\pi}(\alpha_1, 1 - \alpha_1 - \alpha_2, \alpha_2) \left[1 - \frac{m_c^2 - p^2}{(\alpha_1 + u\alpha_2)M^2} \right] + \dots . \quad (14)$$

Here, φ_p , φ_σ , and $\varphi_{3\pi}$ are twist-3 pion wave functions. The ellipses denote contributions of higher twist. The contributions of twist 4 are given explicitly in refs. ^{4,5}. The analogous sum rule for the $B \rightarrow \pi$ form factor is obtained from the above by formally changing $c \rightarrow b$ and $D \rightarrow \bar{B}$.

The numerical values to be substituted for m_c , f_D and s_0 are interrelated by the QCD sum rule for the two-point correlation function $\langle 0 | T\{j_5(x), j_5^+(0)\} | 0 \rangle$, $j_5 = \bar{c}i\gamma_5 u$. We use the set [$f_D = 170 \pm 10$ MeV, $m_c = 1.3$ GeV, $s_0^{(c)} = 6$ GeV²] which satisfies this two-point sum rule without $O(\alpha_s)$ corrections in consistency with the neglect of $O(\alpha_s)$ corrections in the sum rule for $f_D f_D^+$. The uncertainty quoted for f_D corresponds to the variation with the Borel parameter M^2 within the appropriate range of M^2 . A similar interrelation exists for m_b , f_B and $s_0^{(b)}$, where the analogous two-point sum rule yields the set of values [$f_B = 140$ MeV, $m_b = 4.7$ GeV, $s_0^{(b)} = 35$ GeV²]. The variation of f_B with M^2 is negligible.

For the pion wave functions we use the parametrization suggested in ref. [6]. Arguments for this choice are given in ref. [5]. The maximum momentum transfer p^2 at which the sum rule (12) is applicable is estimated to be about 1 GeV² for D mesons and 15 GeV² for B mesons. The resulting form factors $f_D^+(p^2)$ and $f_B^+(p^2)$ are plotted in Fig. 2. The dependence of eq. (12) on the Borel parameter M^2 is rather weak in the range where the twist-4 and the continuum contributions are less than 10% and 30%, respectively [4]. For definiteness, we have taken $M^2 = 4$ GeV² for the $D \rightarrow \pi$ form factor and $M^2 = 10$ GeV² for the $B \rightarrow \pi$ form factor plotted in Fig. 2.

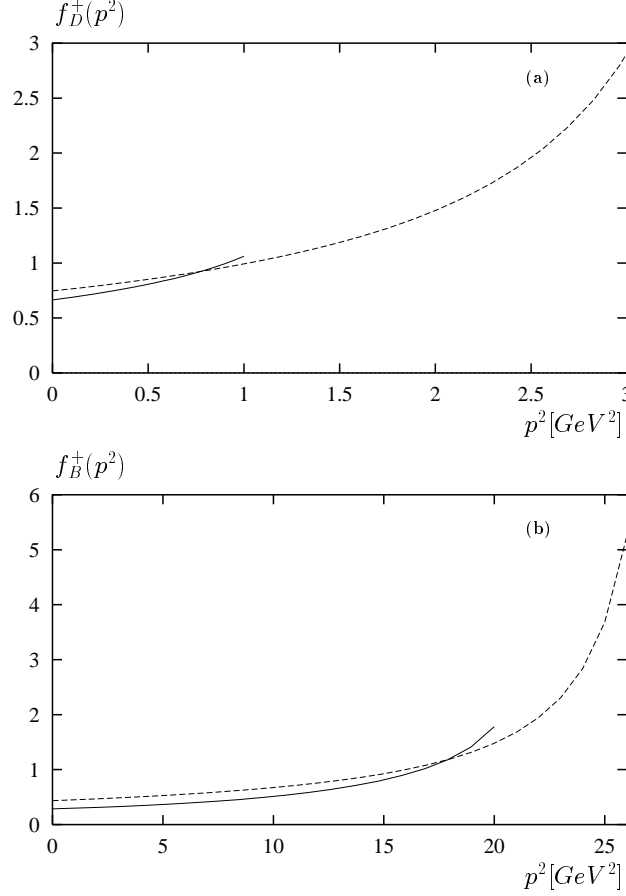


Fig. 2. The form factors for the transitions (a) $D \rightarrow \pi$ and (b) $B \rightarrow \pi$ as predicted by the light-cone sum rule (solid lines) in comparison to the single-pole approximation (dashed lines) with the normalization fixed by the coupling constants $g_{D^*D\pi}$ and $g_{B^*B\pi}$, respectively, determined from the analogous sum rules.

2.3 $D^*D\pi$ and $B^*B\pi$ couplings

Next we sketch how the relation (8) can be turned into a sum rule for the coupling constant $g_{D^*D\pi}$. The key idea is to write a double dispersion integral for the invariant function F :

$$\begin{aligned}
 F(p^2, (p+q)^2) = & \frac{m_D^2 m_{D^*} f_D f_{D^*} g_{D^*D\pi}}{m_c(p^2 - m_{D^*}^2)((p+q)^2 - m_D^2)} \\
 & + \int \frac{\rho^h(s_1, s_2) ds_1 ds_2}{(s_1 - p^2)(s_2 - (p+q)^2)} \\
 & + \int \frac{\rho_1^h(s_1) ds_1}{s_1 - p^2} + \int \frac{\rho_2^h(s_2) ds_2}{s_2 - (p+q)^2}.
 \end{aligned} \tag{15}$$

Here, the first term arises from the ground state contribution and contains the $D^*D\pi$ coupling defined by the on-shell matrix element

$$\langle D^{*+}(p)\pi^-(q) | D^0(p+q) \rangle = -g_{D^*D\pi} q_\mu \epsilon^\mu, \tag{16}$$

while the spectral function $\rho^h(s_1, s_2)$ represents higher resonances and continuum states in the D^* and D channels. The additional single dispersion integrals are due to necessary subtractions. Then, considering p^2 and $(p+q)^2$ as independent variables and applying the Borel operator (11) to eq. (15) with respect to both p^2 and $(p+q)^2$, we obtain

$$F(M_1^2, M_2^2) \equiv \mathcal{B}_{M_1^2} \mathcal{B}_{M_2^2} F(p^2, (p+q)^2) = \frac{m_D^2 m_{D^*} f_D f_{D^*} g_{D^* D \pi}}{m_c} e^{-\frac{m_{D^*}^2}{M_1^2} - \frac{m_D^2}{M_2^2}} + \int e^{-\frac{s_1}{M_1^2} - \frac{s_2}{M_2^2}} \rho^h(s_1, s_2) ds_1 ds_2, \quad (17)$$

where M_1^2 and M_2^2 are the Borel parameters associated with p^2 and $(p+q)^2$, respectively. Note that contributions from heavier states are now exponentially suppressed by factors $\exp\{-\frac{s_{1,2}^2 - m_{D^*,D}^2}{M_{1,2}^2}\}$ as desired, while the subtraction terms depending only on one of the variables, p^2 or $(p+q)^2$, vanish.

Applying the same transformation to the expression (8) and equating the result with eq. (17) we end up with the sum rule

$$\frac{m_D^2 m_{D^*} f_D f_{D^*}}{m_c} \cdot g_{D^* D \pi} = m_c f_\pi \varphi_\pi(u_0) M^2 \exp\left[\frac{m_{D^*}^2 - m_c^2}{M_1^2} + \frac{m_D^2 - m_c^2}{M_2^2}\right] + \dots \quad (18)$$

where $u_0 = \frac{M_1^2}{M_1^2 + M_2^2}$ and $M^2 = \frac{M_1^2 M_2^2}{M_1^2 + M_2^2}$. The ellipses refer to higher-twist and gluonic contributions. The contributions from higher states are again subtracted invoking semilocal duality as discussed in detail in ref. [5].

Since M_1^2 and M_2^2 are expected to be quite similar in magnitude, the coupling constant $g_{D^* D \pi}$ is determined by the value of the pion wave function at $u \simeq 1/2$, that is by the probability for the quark and the antiquark to carry equal momentum fractions in the pion. This interesting feature is shared by the sum rules for many other important hadronic couplings involving the pion. As already pointed out, the quantity $\varphi_\pi(1/2)$ is considered to be a universal nonperturbative parameter, similar to quark and gluon condensates in the standard approach. It may be determined from suitable sum rules in which the phenomenological part is known experimentally. We take the value $\varphi_\pi(1/2) = 1.2 \pm 0.2$ obtained from the light-cone sum rule for the pion-nucleon coupling [6]. For the remaining parameters we use the same input values as in the calculation of the form factor f_D^+ in Section 2.2. In addition, we take $f_{D^*} = 240 \pm 20$ MeV as determined from the corresponding two-point sum rule. With this choice, we obtain

$$g_{D^* D \pi} = 12.5 \pm 1.0. \quad (19)$$

The uncertainty indicates the variation of $g_{D^* D \pi}$ in the interval $2 \text{ GeV}^2 < M^2 < 4 \text{ GeV}^2$, where the higher state contributions are less than 30% and the twist-4 corrections do not exceed 10%. The sensitivity to the effective threshold s_0 is reasonably small. For example, variation of $s_0^{(c)}$ between 5 and 7 GeV^2 , while all other parameters are kept fixed, leads to a total variation of the coupling $g_{D^* D \pi}$ by less than 5%. The above prediction can be directly tested experimentally in the decay $D^* \rightarrow D \pi$. Eq. (19) implies the decay width $\Gamma(D^{*+} \rightarrow D^0 \pi^+) = 32 \pm 5 \text{ keV}$, which is well below the current experimental upper limit [9, 10] $\Gamma(D^{*+} \rightarrow D^0 \pi^+) < 89 \text{ keV}$.

The sum rule for $g_{D^* D \pi}$ given in eq. (18) is easily converted into a sum rule for the coupling $g_{B^* B \pi} = g_{\bar{B}^0 B^- \pi^+}$ by replacing c with b , D with \bar{B} , and D^* with \bar{B}^* . Using $f_{B^*} = 160$ MeV in addition to the B -channel parameters specified in Section 2.2 and confining oneself to the corresponding fiducial interval $6 \text{ GeV}^2 < M^2 < 12 \text{ GeV}^2$, one finds

$$g_{B^* B \pi} = 29 \pm 3. \quad (20)$$

If the threshold $s_0^{(b)}$ is varied between 34 and 36 GeV², this value changes by 5%.

The dependence on the pion wave function disappears in the limit $q \rightarrow 0$ as can be seen from eq. (8) because of the normalization condition $\int_0^1 du \varphi_\pi(u) = 1$. This is just the limit where the correlation function (1) can be treated in short-distance expansion. The condition $q \simeq 0$ is also implicitly assumed in refs. [7, 8] where the correlation function (1) is calculated using the external field method, or equivalently the soft-pion approximation. Our more general calculation [5] confirms the result of ref. [8].

3 Pole Model for $D \rightarrow \pi$ and $B \rightarrow \pi$ form factors

The couplings $g_{D^*D\pi}$ and $g_{B^*B\pi}$ fix the normalization of the form factors of the heavy-to-light transitions $D \rightarrow \pi$ and $B \rightarrow \pi$, respectively, in the pole-model description [11]:

$$f_D^+(p^2) = \frac{f_{D^*} g_{D^*D\pi}}{2m_{D^*}(1 - p^2/m_{D^*}^2)} . \quad (21)$$

An analogous expression holds for the form factor $f_B^+(p^2)$.

It is difficult to justify the pole model from first principles. Generally, it is believed that the vector dominance approximation is valid at zero recoil, that is at $p^2 \rightarrow m_D^2$. Arguments based on heavy quark symmetry suggest a somewhat larger region of validity characterized by $(m_D^2 - p^2)/m_c \sim O(1\text{GeV})$. However, there are no convincing arguments in favour of this model to be valid also at small values of p^2 which are most interesting from a practical point of view. Nevertheless, using the results presented in Sections 2.2 and 2.3 one observes that not only the shape but also the absolute normalization of the form factors at low p^2 appears to be in rough agreement with the pole model. This is illustrated in Fig. 2. Quantitatively, at $p^2 = 0$ we find $f_D^+(0)_{SR} = 0.66$, $f_D^+(0)_{PM} = 0.75$, and $f_B^+(0)_{SR} = 0.29$, $f_B^+(0)_{PM} = 0.44$. In the regions $m_Q^2 - p^2 > O(1\text{ GeV}^2)$ with $Q = c$ and b , respectively, the numerical agreement between the light-cone sum rule and the pole model is better than 15% for f_D^+ , and still within 50% for f_B^+ . This finding is surprising. Even if the contributions of several low-lying resonances in the D^* (B^*) channel may mimic the p^2 dependence of a single pole, there is no reason for the normalization to be mainly given by the coupling $g_{D^*D\pi}$ ($g_{B^*B\pi}$) to a good (rough) approximation.

Despite of the overall qualitative agreement in the mass range of D and B mesons, the light-cone sum rule and the pole-dominance model differ markedly in the asymptotic dependence of the form factors on the heavy mass. Focusing on B mesons and using the familiar scaling laws $f_B\sqrt{m_B} = \hat{f}_B$, $f_{B^*}\sqrt{m_B} = \hat{f}_{B^*}$ and $g_{B^*B\pi} = (2m_B/f_\pi)\hat{g}$ which are expected to be valid at $m_b \rightarrow \infty$ modulo logarithmic corrections, the pole model predicts $f_B^+(0)_{PM} \sim 1/\sqrt{m_B}$, whereas the light-cone sum rule (12) yields $f_B^+(0)_{SR} \sim 1/m_B^{3/2}$. The latter result rests on the behaviour in QCD of the leading twist pion wave function near the end point, that is on $\varphi_\pi(u) \sim 1 - u$ at $u \rightarrow 1$.

Since we see no theoretical justification for extrapolating the pole model to the region $p^2 = 0$ we believe the sum rule result. The solution suggested by Fig. 2 is then to match the two descriptions in the region of intermediate momentum transfer $p^2 \simeq m_Q^2 - O(1\text{GeV}^2)$. Referring for a detailed discussion to ref. [5] we emphasize that the light-cone sum rules seem to be generally consistent with the heavy quark expansion. In particular, the light-cone sum rule (18) correctly reproduces the heavy quark mass dependence of the coupling $g_{B^*B\pi}$. Fitting

our predictions for $g_{B^*B\pi}$ and $g_{D^*D\pi}$ to the form

$$g_{B^*B\pi} = \frac{2m_B}{f_\pi} \cdot \hat{g} \left[1 + \frac{\Delta}{m_B} \right] \quad (22)$$

and the analogous expression for $g_{D^*D\pi}$, we find for the coupling \hat{g} and the strength Δ of the $1/m_Q$ correction:

$$\hat{g} = 0.32 \pm 0.02, \quad \Delta = (0.7 \pm 0.1) \text{ GeV}. \quad (23)$$

4 Nonfactorizable effects in the decay $B \rightarrow J/\psi K$

Nonleptonic two-body decays of heavy mesons are usually calculated by factorizing the appropriate matrix element of the weak Hamiltonian H_W into a product (or a sum of such products) of a form factor and a decay constant. However, as well known, naive factorization fails. In order to achieve agreement with experiment it is necessary to let the Wilson coefficients $a_{1,2}$ emerging from the operator product expansion of H_W and multiplying the relevant weak matrix elements deviate from the values predicted in short-distance QCD. Phenomenologically [12], $a_{1,2}$ are treated as free parameters to be determined from experiment.

The decay $B \rightarrow J/\psi K$ provides an important example. The relevant part of the weak effective Hamiltonian may be written as

$$H_W = \frac{G}{\sqrt{2}} V_{cb} V_{cs}^* \{ (c_2 + \frac{c_1}{3}) O_2 + 2c_1 \tilde{O}_2 \}, \quad (24)$$

with the four quark operators

$$O_2 = (\bar{c}\Gamma^\rho c)(\bar{s}\Gamma_\rho b), \quad \tilde{O}_2 = (\bar{c}\Gamma^\rho \frac{\lambda^a}{2} c)(\bar{s}\Gamma_\rho \frac{\lambda^a}{2} b) \quad (25)$$

and $\Gamma_\rho = \gamma_\rho(1 - \gamma_5)$. In factorization approximation, the decay amplitude is given by

$$\langle J/\psi(p) K(q) | H_W | B(p+q) \rangle = \sqrt{2} G V_{cb} V_{cs}^* a_2 f_\psi f_K^+ m_\psi (\epsilon^\psi \cdot q) \quad (26)$$

where $a_2 = c_2 + \frac{c_1}{3}$, f_ψ is the decay constant of the J/ψ , f_K^+ is the $B \rightarrow K$ form factor at $p^2 = m_\psi^2$, and ϵ^ψ denotes the J/ψ polarization vector. From the short-distance value of a_2 , the branching ratio is estimated to be almost an order of magnitude smaller than the experimental result [13, 14]. On the other hand, dropping the term proportional to $c_1/3$ in a_2 as suggested in the framework of the $1/N_c$ expansion [15] of the weak amplitudes yields reasonable agreement. One can argue that the factorizable term proportional to $c_1/3$ is cancelled by nonfactorizable contributions being of the same order in $1/N_c$. Such a cancellation was first advocated in ref. [15] and then shown in ref. [16] to actually take place in two-body D decays. In the latter work QCD sum rule techniques were used in order to estimate the nonfactorizable amplitudes.

Recently, we have investigated the problem of factorization in B decays using $B \rightarrow J/\psi K$ as a study case [17]. Following the general idea put forward in ref. [16], we calculate the four-point correlation function

$$< 0 | T \{ j_{\mu 5}^K(x) j_\nu^\psi(y) H_W(z) j_5^B(0) \} | 0 > \quad (27)$$

by means of the short-distance OPE. Here $j_{\mu 5}^K = \bar{u}\gamma_\mu\gamma_5 s$, $j_\nu^\psi = \bar{c}\gamma_\nu c$ and $j_5^B = \bar{b}i\gamma_5 u$ are the generating currents of the mesons involved and H_W is the effective weak Hamiltonian (24).

To lowest nonvanishing order in α_s the nonfactorizable contributions to the matrix element (27) only arise from the operator \tilde{O}_2 in H_W . Obviously, the contribution of this operator to the $B \rightarrow J/\psi K$ amplitude (26) vanishes by factorization because of colour conservation. Parametrizing the nonfactorizable matrix element by

$$\langle J/\psi(p)K(q) | \tilde{O}_2 | B(p+q) \rangle = 2\tilde{f}f_\psi m_\psi (\epsilon^\psi \cdot q) \quad (28)$$

we construct a sum rule for \tilde{f} which enters the correlation function (27) through the ground state contribution. In the QCD part of this sum rule all nonperturbative contributions from vacuum condensates up to dimension 6 are included. The corresponding diagrams are indicated in Fig. 3. In the hadronic part a complication arises from intermediate states in the B -meson channel carrying the quantum numbers of a $\bar{D}D_s^*$ pair. These virtual states are created by weak interaction and converted into the $J/\psi K$ final state by strong interaction. In the quark-gluon representation of the correlation function (27) calculated from the diagrams of Fig. 3 one can identify corresponding four-quark $\bar{u}s\bar{c}c$ intermediate states. Invoking quark-hadron duality we cancel this piece of the QCD part against the unwanted hadronic contribution.

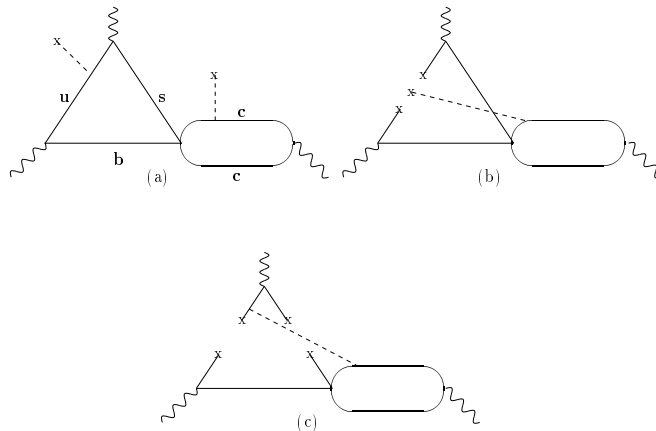


Fig. 3. Diagrams associated with (a) the gluon condensate, (b) the quark-gluon condensate and (c) the four-quark condensate contributions to the correlation function (27) with H_W replaced by \tilde{O}_2 .

We then perform, as usual, a Borel transformation in the B -meson channel and take moments in the charmonium channel. The spacelike momentum squared in the K -meson channel is kept fixed. As explained in ref. [17], at this stage one encounters a second problem. The usual subtraction of higher state contributions employing semilocal quark-hadron duality is not possible here. Therefore, we must include these contributions explicitly in the sum rule. For this purpose we use a simple two-resonance model for the spectral functions in each of the three channels: B and B' in the $\bar{u}b$ -channel, J/ψ and ψ' in the $\bar{c}c$ -channel, and K and K' in the $\bar{u}s$ -channel. This rough approximation, yields $\tilde{f} = -(0.045 \div 0.075)$. The full decay amplitude for $B \rightarrow J/\psi K$ is proportional to

$$a_2 = c_2 + \frac{c_1}{3} + 2c_1\tilde{f}/f_K^+, \quad (29)$$

where the first two coefficients are associated with the factorizable part of the matrix element (26), while the third term is due to the leading nonfactorizable term (28). Interestingly enough, we find that the factorizable nonleading in $1/N_c$ term $c_1/3$ and the nonfactorizable term in (29) are opposite in sign. Although the nonfactorizable matrix element is considerably smaller than the factorizable one, $|\tilde{f}/f_K^+| \simeq 0.1$, it has a strong quantitative impact due to its large coefficient, $|2c_1/(c_2 + c_1/3)| \simeq 20 \div 30$. In fact, if $|\tilde{f}|$ is close to the upper end of the predicted range, the third term in eq. (29) almost cancels the second term, thereby increasing the branching ratio considerably. This is exactly the scenario anticipated by $1/N_c$ -rule [15].

It is also very interesting to note that our theoretical estimate yields a negative overall sign for a_2 in contradiction to a global fit to data [14]. Furthermore, there is no theoretical reason in our approach to expect universal values or even universal signs for the coefficients $a_{1,2}$ in different channels, in contrast to what seems to be suggested by experiment. Universality can at most be expected for certain classes of decay modes, such as $B \rightarrow D\pi$ or $B \rightarrow D\overline{D}$, etc. Also, there is no simple relation between B and D decays in our approach since the OPE for the corresponding correlation functions significantly differ in the relevant diagrams and in the hierarchy of mass scales. We hope to be able to clarify these issues further.

Concluding we would like to stress that QCD seems to predict a much richer pattern in two-body weak decays than what is revealed by the current phenomenological analysis of the data.

5 Conclusion

The flexible and careful employment of QCD sum rule techniques in the analysis of exclusive heavy meson decays promises considerable progress in solving the open problems, at least some of them.

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